

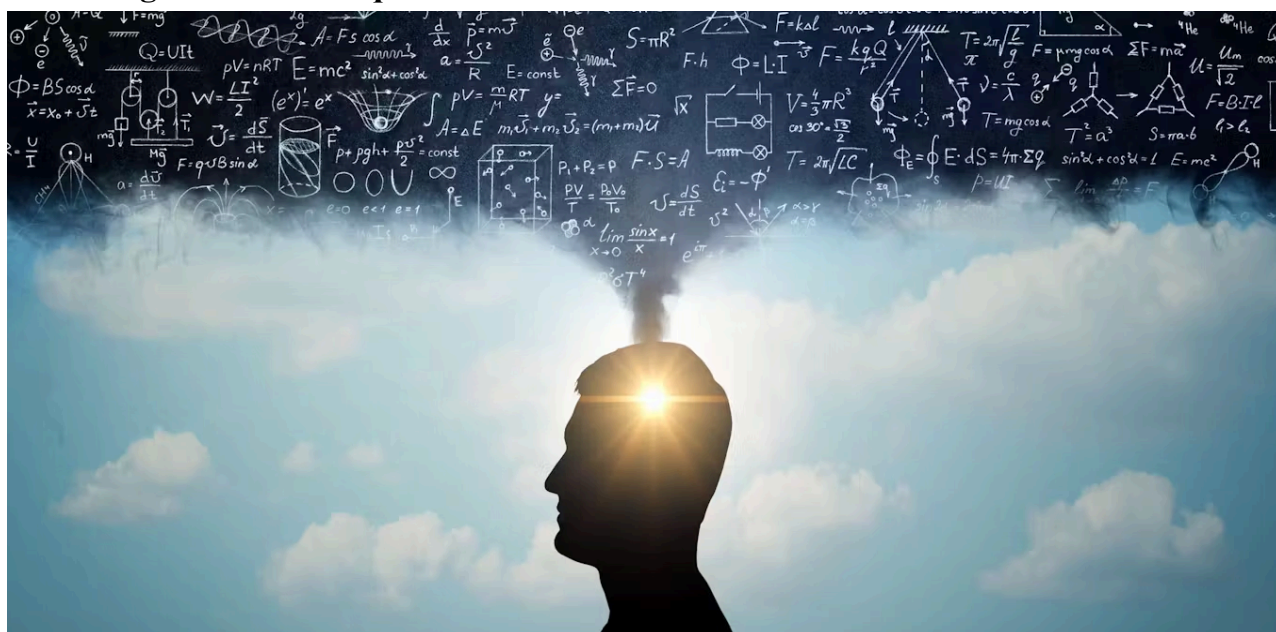
## The Calculus Of Chaos

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Q5: To what extent can mathematical modelling capture  
the complexity of human behaviour?

## Modelling the Human Equation



### Introduction

Suppose you're asked to pick a number between 0 and 100, with the goal of choosing the number closest to two-thirds of the average of all guesses. What number would you pick? A perfectly rational, hyper-mathematical actor would reason recursively: if everyone is rational, the average will be 50, so I should choose 33. But if everyone thinks that, the average becomes 33, so I should choose 22. This logic iterates all the way down to 0. Yet, in the real world, the winning number is never zero. It clusters somewhere between 20 and 25. This "Guess  $\frac{2}{3}$  of the Average" game perfectly encapsulates the central challenge

of economic modelling. This essay will argue that while mathematical models are indispensable tools for isolating fundamental forces and making quantitative predictions, their ability to capture the full complexity of behaviour is inherently bounded.

### Rational Choice Theory

Mathematical models excel not by replicating reality in its entirety, but by stripping it down to its core components. Consider the canonical model of rational choice: an individual is presented with a set of consumption bundles,  $X = (x_1, x_2, \dots, x_n)$ . They are assumed to have a stable, well-defined preference ordering that is

complete and transitive. Their behaviour is modelled as the solution to a constrained optimisation problem:

$$\max_{x \in X} U(x) \quad \text{subject to} \quad p \cdot x \leq m$$

where  $U(x)$  is a utility function representing their preferences,  $\mathbf{p}$  is a vector of prices, and  $\mathbf{m}$  is their income. The solution to this problem yields a demand function,  $x^*(\mathbf{x}_1, \mathbf{x}_2)$ , which predicts how choices change with prices and income. We can visualize this relationship with a consumer choice diagram (Fig. 1). The budget constraint, defined by prices and income, outlines the set of all affordable bundles. The utility function is represented by indifference curves, each showing combinations of goods that provide equal satisfaction. The optimal choice  $(\mathbf{x}_1, \mathbf{x}_2)$  is found where the highest attainable indifference curve is tangent to the budget constraint, or where the Marginal Rate of Substitution (MRS) is equal to the Marginal Rate of Transformation (MRT). (Varian, 2010)

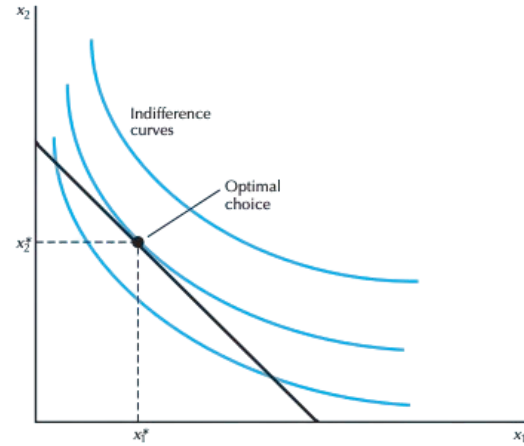


Figure 1: The consumer choice model with indifference curves and a budget constraint. (Source: Adapted from standard microeconomic theory, e.g., Varian, 2014)

The true analytical power of this framework is revealed when prices change. The Slutsky equation, derived mathematically from this model, neatly decomposes a price change's effect into a substitution effect (the pure response to changing relative prices) and an income effect (the change in purchasing power):

$$\frac{\partial x_i}{\partial p_j} = \frac{\partial h_i}{\partial p_j} - x_j \frac{\partial x_i}{\partial m}$$

where  $\mathbf{h}_i$  is the Hicksian (compensated) demand. This mathematical decomposition allows policymakers to forecast, for instance, the impact of a sugar tax, distinguishing between consumers switching to alternatives (substitution) and simply reducing overall consumption (income). In controlled environments or for aggregate,

predictable behaviours like basic consumption, this modelling approach is remarkably powerful and provides the foundational language of modern economics.

## The Ultimatum Game

However, the “Guess  $\frac{2}{3}$  of the Average” game reminds us that humans are not bound by simple models, instead, human behaviour is complex and deeply influenced by our belief about others. The consumer choice model provides a baseline, but it operates on the assumption of perfect rationality which almost always unravels when confronted with the complexities of the human mind. We can study these limitations of mathematical models by exploring the classic Ultimatum Game.

In this experiment, one participant (the Proposer) is endowed with a sum of money and must propose a division for a second participant (the Responder). If the Responder accepts, the split is paid out. If they reject, both players receive nothing. The rational choice model predicts a straightforward outcome: the Proposer, aiming to maximize their own payoff, will offer the smallest possible positive amount. The Responder, presented with a choice

between something and nothing, should accept it. Figure 2 maps this logic as a game tree. At each branch, the Proposer chooses how much to offer, and the Responder chooses whether to accept (A) or reject (R). The payoffs at the end of each branch follow straightforwardly which shows acceptance yields the proposed split and rejection leaves both with nothing. Backward induction applied to this tree predicts a simple outcome that the Responder should accept any positive offer, and anticipating this, the Proposer should offer the smallest possible amount.

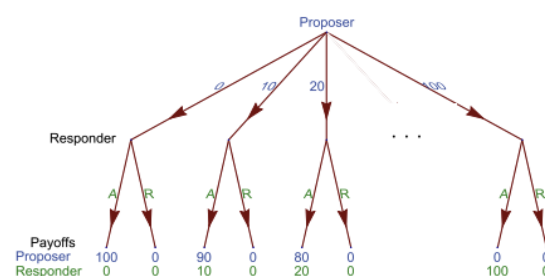


Figure 2: Game tree of the Ultimatum Game. The Proposer chooses an offer, and the Responder either accepts (A) or rejects (R) (Source: The Ultimatum Game, R. Root, 2019)

However, across countless replications, this is not what occurs. Proposers rarely make such minimal offers, and Responders frequently reject them when they do. Instead, offers commonly cluster around 40-50% of the total, with the 50-50 split often emerging as the modal outcome (Camerer, 2023). Additionally, low offers

(below about 20%) are systematically punished, even at personal cost to the Responder. The Ultimatum Game cannot be explained by a standard utility function,  $U(x)$ , based solely on personal consumption. As Farmer and Geanakoplos (2008) argue, economic models can be insightful, but their limitations become apparent in economies subject to feedback effects, nonlinear interactions, and adaptive expectations.

Nonetheless, economists can expand the framework by introducing new variables. For example, incorporating social preferences such as  $U(\pi_i, \pi_j)$ , where an individual's utility depends on both their own payoff ( $\pi_i$ ) and the payoff of others ( $\pi_j$ ), would make it appear closer to real behaviour

But at the end of the day, unlike prices and quantities, emotions like resentment and fairness (as seen in the Ultimatum Game) are not easily reduced to equations.

## Conclusion

In conclusion, the journey through mathematical models, from the rational consumer choice model to the complex behavioural dynamics of the Ultimatum Game, reveals a nuanced answer.

Mathematical modelling does not so much “capture” the complexity of human behaviour as it “engages” with it at different levels.

From a self-driving car's algorithm predicting a pedestrian's likely path based on historical data, to a central bank forecasting market reactions of an interest rate hike, models can stretch far, but they eventually fall short when confronted with the full spectrum of human emotion and social interactions.

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